COMPRESSIVE SENSING OF A SUPERPOSITION OF PULSES

Chinmay Hegde and Richard G. Baraniuk

Rice University

Compressive Sensing (CS)

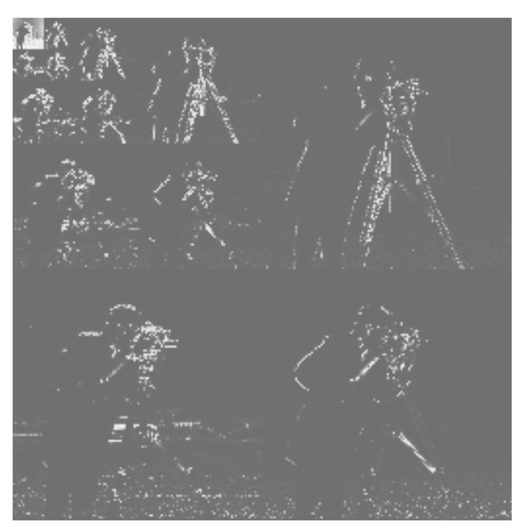
- Natural/manmade signals often have **sparse/compressible** structure
- Traditional signal acquisition: sample first, then compress
- Compressive sensing: sample and compress simultaneously

Sparsity and Compression

Traditional signal acquisition:

- *Sample* data at Nyquist rate (2x bandwidth)
- *Compress* data (signal dependent, nonlinear)

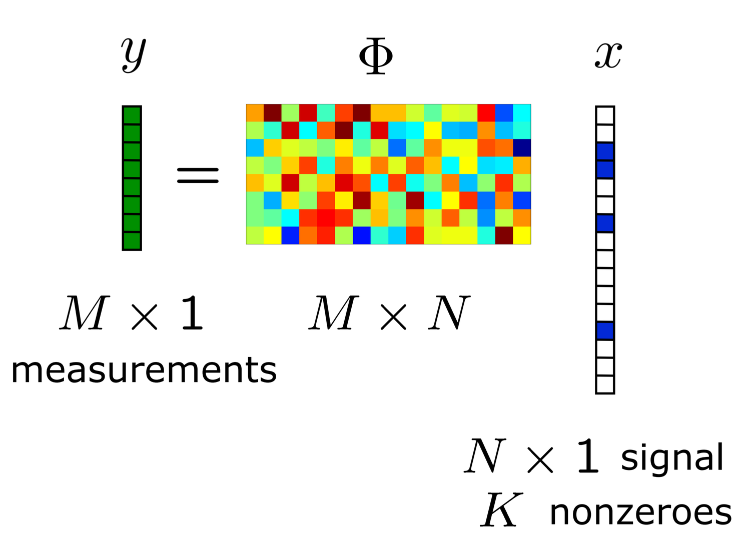
N pixel image

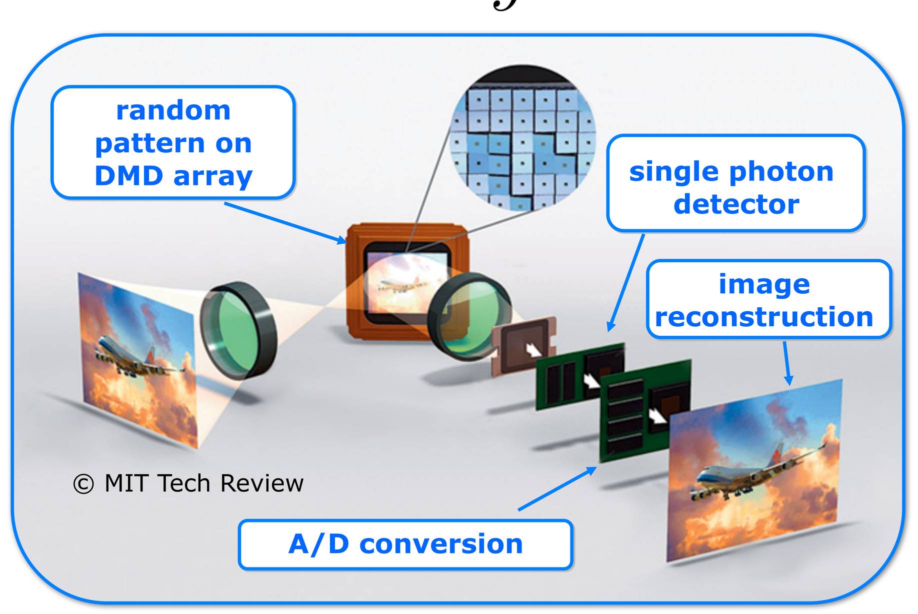


K large wavelet coefficients

Compressive Sensing (CS)

- Acquire *compressive measurements* $y = \Phi x$





Rice Compressive Imaging Camera

$$M \geq O(K \log(N/K))$$

Signal Recovery

- Recovery algorithms *rely on sparsity*
  - ℓ_1 minimization (slow, strong guarantees for recovery)
  - orthogonal matching pursuit (fast, weak guarantees)
  - CoSaMP / IHT (fast, strong guarantees)

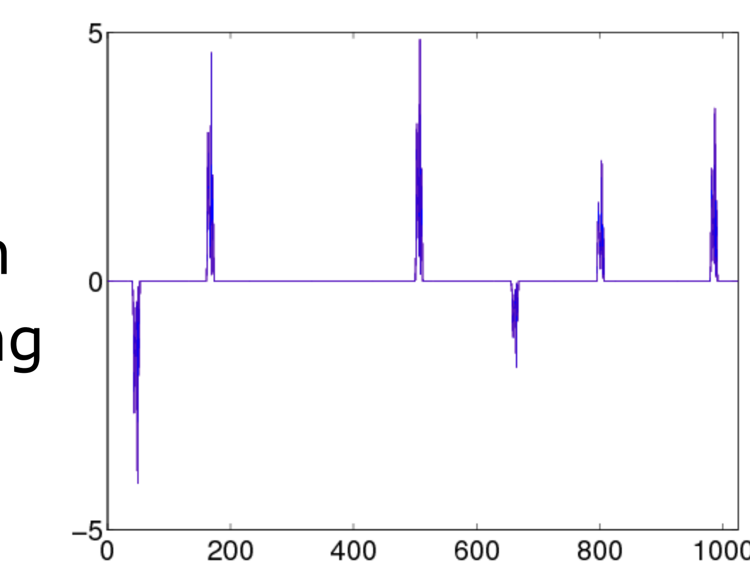
Structured Sparsity

- Sparsity assumption does not capture **dependencies** among coefficients
- New model for signals well-approximated by a sparse sum of pulses
- Provably reduces number of measurements needed to sample signals

Sparse Sums of Pulses

Sparsity is often an oversimplification

- 1D example: pulse stream
  $N = 1024, K = 66$
- Signal consists of $S = 6$ pulses of width $F = 11$ with identical shapes but varying amplitudes and locations
- Can we exploit this special structure in CS recovery?



Proposed Signal Model

Signals of interest can be written as

$$z = x * h = Hx = Xh$$

where:

- $x \in \mathcal{M}_S^\Delta$, the space of all S-sparse images with nonzeroes separated by at least Δ locations
- $h \in \mathcal{M}_\Omega$, the space of all minimum phase filters defined over a domain Ω
- Proposed model: *Infinite union of subspaces*

Sampling Theorem

$$M \geq O\left((S + |\Omega|) + S \log(N/S - \Delta)\right)$$

- Overall number of nonzeroes: $K = S|\Omega|$
- Hence, number of measurements is *sublinear* in the sparsity K

Improved CS Recovery

- Requires far fewer measurements than state-of-the-art CS methods
- Recovery robust under noise, model mismatch
- Testing performed on synthetic and real data

Iterative support estimation + deconvolution

Input: measurements $y = \Phi z$, matrix Φ

Output: Estimates $\widehat{x}$, $\widehat{h}$

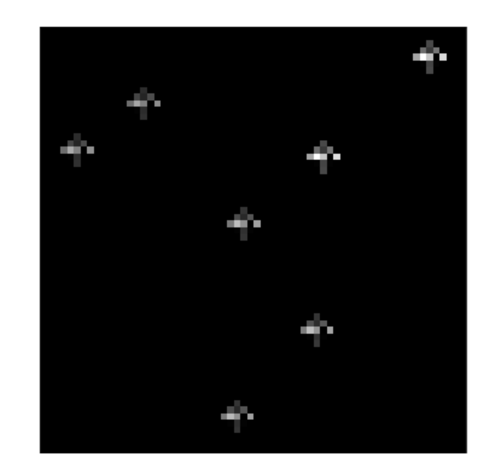
Initialize: $\widehat{H} \leftarrow I$

Repeat until convergence:

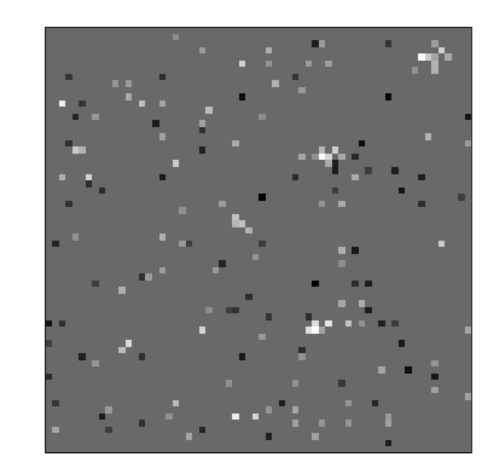
- Solve for $\widehat{x}$ via model-based CoSaMP: $y = \Phi \widehat{H} \widehat{x}$
- Solve for $\widehat{h}$ via pseudoinverse: $y = \Phi \widehat{X} \widehat{h}$
- Update estimate of signal: $\widehat{z} \leftarrow \widehat{x} * \widehat{h}$

Synthetic test image

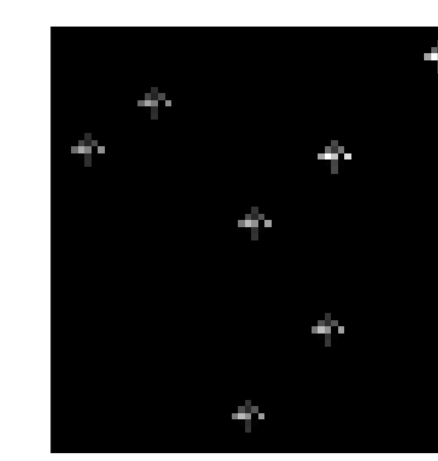
$$N = 64 \times 64, S = 7, |\Omega| = 25, M = 290$$



Test image



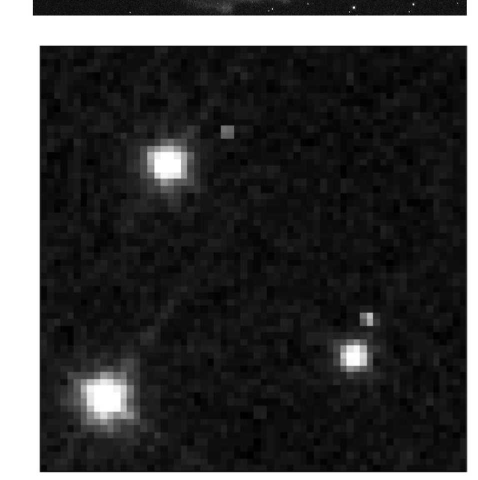
CoSaMP (MSE = 16.95)



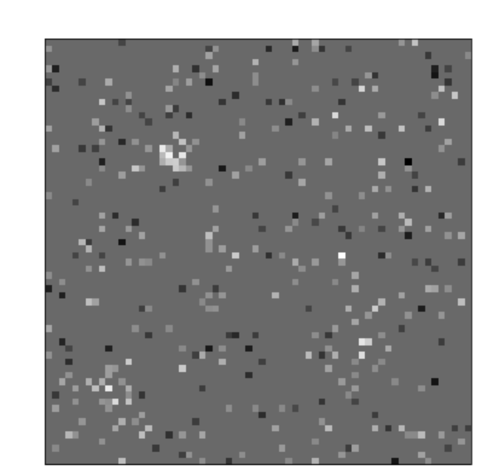
New algorithm (MSE = 0.07)

Real-world test image

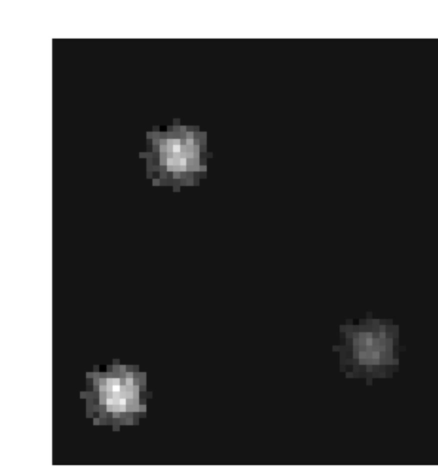
$$N = 64 \times 64, S = 3, |\Omega| = 120, M = 330$$



Test image



CoSaMP



New algorithm